XVIII. *On Improvements in the Instruments and Methods employed in determining the direction and intensity of the Terrestrial Magnetic Force.* By S. Hunter Christie, *Esq. M.A. F.R.S. M.C.P.S. Soc. Philom. Paris. Corresp., &c.*

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All who have had occasion to make observations with a dipping needle, are aware of the tedious nature of those observations, and of the uncertainty which frequently attends the results, even when the best instruments that can be procured are employed. The observations by which the terrestrial magnetic intensity is determined are more tedious, and subject even to greater uncertainty. Having long been sensible of the imperfections of the instruments by which the dip is determined, the improvement of them has at different times engaged my attention; and various methods have suggested themselves by which, theoretically, this might be effected, but practical difficulties, at the same time, presented themselves to these proposed improvements.

The uncertainty attending dip observations arises principally from two sources; the want of perfect freedom of motion on the axis, and the non-coincidence of the centre of gravity of the needle with the axis of motion, the latter rendering necessary the inversion of the poles of the needle. It is now many years since it occurred to me, that increased freedom of motion might be obtained by making the axis of the needle a very acute double cone, instead of a cylinder, and inclining the agate planes at a greater angle to the horizon than half the angle of each cone: but it appeared to me also, that a degree of horizontal pressure on the axis would arise from the inclination of the supporting planes, which might counterbalance the advantages arising from the diminution of the parts in contact; and, besides, that there would be considerable difficulty in adjusting the axis of the needle so as to be perpendicular to the plane of the graduated circle, or truly horizontal. These considerations induced me to abandon the idea of such a construction. If, however, I am correctly informed, for I have not yet seen the instrument, the dipping needles now made by GAMBEY are nearly on this construction; and if so, the practical difficulties which occurred to me have been overcome by him. I have also recently seen a needle on nearly the same principle, which must have been made before GAMBEY'S, since Captain KING employed it during his survey of the sea to the north of New Holland: so little idea, however, had the maker of diminishing friction by this means, that the conical ends of the axis of the needle rested in conical caps of brass, until, at my suggestion, agate caps were substituted. By such means, then, the friction on the axis may very probably be diminished, but the difficulty respecting the position of the centre of gravity still remains.

Some years since, Mr. DOLLOND constructed a needle by which it was proposed to determine the dip without inverting the poles; the errors arising from the excentricity of the centre of gravity being supposed to counterbalance each other, by observing the inclination with the axis of support successively in different positions round the axis of the very acute cones forming the needle. As far, however, as the observations with this needle have come under my notice, they are so widely different from those made at the same places with needles whose poles were inverted, that I greatly suspect the accuracy of the results. Independently of this, the length of time required for the observations is a serious objection to the use of this instrument. Various methods had occurred to me by which the necessity of inverting the poles might be avoided; such as by employing a compound needle, consisting of two needles, moveable about an axis; so that by making observations with the needles separately, and also jointly, in different relative positions, the position of the centre of gravity of each needle, and of the two combined, might be determined: by adjusting small weights to the ends of the needle, by which means the position of the centre of gravity might be adjusted: also by giving two or more axes of suspension to the needle But these methods either led to complicated results, or there appeared practical difficulties in their application. Being, therefore, dissatisfied with them, I had given up the inquiry until very recently, when my attention was again drawn to the subject, by having been consulted respecting the magnetical instruments to be employed on different expeditions, particularly that into the interior of Africa, and that under the direction of Captain BACK. I now consider that no practical difficulties will occur in the construction of a dipping needle on a principle that shall render the inversion of the poles unnecessary, and that a further advantage of such a needle will be, that the observations which give the dip will also give a measure of the terrestrial magnetic intensity, so long as the magnetism of the needle remains unchanged.

Whatever may be the position of the centre of gravity of the needle, there are means by which that position may be determined, previously to the needle being magnetized, and, indeed, after it has undergone that operation. I therefore assume it to be known.

Let, then, g be the distance of the centre of gravity of the needle from the axis of the cylinder or cone at right angles to the plane of the needle forming the axis of support; γ the angle which this distance makes with the magnetic axis of the needle; w the weight of the needle; let δ be the dip, or the angle which the terrestrial magnetic force makes with the horizon; m the force which, acting at the distance $\frac{1}{2}\,l$ from the centre of the needle, is equivalent to the action of the terrestrial magnetism upon the magnetism of each arm of the needle, so that $l\,m$ is the static momentum of the terrestrial magnetism acting upon the magnetism of the needle; also let ${}_{,}\theta$ be the angle, measured from north downwards, which the axis of the needle makes with the horizon, when the centre of gravity of the needle is downwards, and ${}'\theta$ the corresponding angle when the centre of gravity is upwards: then, whether the supporting axis is considered as a line or as a cylinder, the condition of equilibrium will give the two equations,

$$w\,g\cos({}_{,}\theta+\gamma)-l\,m\sin({}_{,}\theta-\delta)=0\,; \qquad (1)$$

$$w\,g\cos({}'\theta-\gamma)-l\,m\sin({}'\theta-\delta)=0. \qquad (2)$$

From these,

$$\tan\delta=\frac{2\sin{}'\theta\sin{}_{,}\theta-\sin({}'\theta-{}_{,}\theta).\cot\gamma}{\sin({}'\theta+{}_{,}\theta)}, \qquad (a)$$

or

$$\tan\delta=\frac{2-(\cot{}_{,}\theta-\cot{}'\theta).\cot\gamma}{\cot{}_{,}\theta+\cot{}'\theta}. \qquad (b)$$

δ being thus known, we shall have,

$$\frac{l\,m}{w\,g}=\frac{\cos({}_{,}\theta+\gamma)}{\sin({}_{,}\theta-\delta)}, \qquad (c)$$

or

$$\frac{l\,m}{w\,g}=\frac{\cos({}'\theta-\gamma)}{\sin({}'\theta-\delta)}\,; \qquad (d)$$

either of which will be a measure of the terrestrial intensity.

If the value of γ has not been determined previously to the needle being magnetized, it may be determined by inverting the poles of the needle; for when the poles are inverted, two other equations are obtained:

$$w\,g\cos(\theta' + \gamma) + l\,m'\sin(\theta' - \delta) = 0, \quad \ldots\ldots \quad (3)$$

$$w\,g\cos(\theta_{,} - \gamma) + l\,m'\sin(\theta_{,} - \delta) = 0, \quad \ldots\ldots \quad (4)$$

where θ' and $\theta_{,}$ are the values of ${}_{,}\theta$ and θ' when γ in the equations (1) and (2) is increased by 180°, and m' may or may not be equal to m. From these equations,

$$\tan\delta = \frac{2 - (\cot\theta' - \cot\theta_{,}).\cot\gamma}{\cot\theta' + \cot\theta_{,}}. \quad \ldots\ldots \quad (e)$$

Eliminating cot γ from the equations (b) and (e),

$$\tan\delta = \frac{\cot{}_{,}\theta + \cot\theta_{,} - (\cot{}'\theta + \cot\theta')}{\cot{}_{,}\theta\cot\theta_{,} - \cot{}'\theta\cot\theta'}. \quad \ldots\ldots \quad (f)$$

The value of δ being obtained from this equation, the value of γ may be obtained from either of the equations (b) or (e). The angle γ being then determined, either previously to the needle being magnetized, or afterwards by inverting its poles, the dip may, on all subsequent occasions, be determined without having recourse to this process, provided the position of the axis of the needle has been permanent.

The equation (f) is fundamentally the same as that by which the dip is determined from observations with Mayer's needles, but in a form more convenient for computation; but I am not aware that it has ever been proposed to determine the dip by means of these needles, without inverting the poles. Captain Sabine, who seems to have entertained a high opinion of needles of this construction, and who obtained with them extremely accordant results, does not appear to have suspected that the inversion of the poles was unnecessary, after having once ascertained the effect of that operation*. It may be objected to the method I have proposed, that whatever change takes place

* Nearly the whole of this paper had been written before I had an opportunity of consulting Mayer's Memoir, "De Usu accuratiori Acus inclinatoriæ Magneticæ," (Comment. Soc. Reg. Scient. Gotting. 1816,) and I was not aware how nearly some of my views coincided with his. He states that two observations, without inverting the poles, will determine the dip, when it has been ascertained, previously to magnetizing the needle, that its centre of gravity is in the line perpendicular to the axis of the needle and the axis of support. When this is not the case, the dip is determined by four observations, the poles of the needle in the second pair being inverted; and he does not deter-

in the magnetic axis of the needle, in this single operation of inverting the poles, for the purpose of determining γ, will affect the results of all the future observations, since it will affect the value of γ, which will enter in all those results. I am aware of the validity of this objection, and would only have recourse to the method to remedy an omission in the original construction of the needle. But allowing full weight to the objection, it is to be remembered that this operation having to be performed but once, every possible precaution may be taken, in magnetizing the needle, to prevent any deviation of the magnetic axis from the axis of figure; whereas, if the dip is to be determined on

mine the value of the angle γ from the observations which he gives as examples, so that this value might be applied to future observations, without again having recourse to the inversion of the poles. He states that the angle γ may be determined without performing this operation; but I consider that he drew this conclusion without having resolved the equations on which he makes that determination to depend, since those equations will not determine it. He proposes that the instrument should be placed so that the plane of vibration of the needle should be at right angles to the magnetic meridian. and the angles which the needle makes with the vertical being observed, will serve for the determination of γ. Let ${}_{,}\varphi$ and ${}'\varphi$ be the angles which the needle makes with the horizon, when its centre of gravity is downwards, and when the needle is reversed on the axis, in this position of the instrument, these angles being measured from the same point; then the condition of equilibrium will give the two equations,

$$w\,g \cos({}_{,}\varphi + \gamma) + l\,m \sin\delta \cos{}_{,}\varphi = 0,$$
$$w\,g \cos({}'\varphi - \gamma) + l\,m \sin\delta \cos{}'\varphi = 0,$$

which are equivalent to those given by MAYER, though our notations are different.

From these two equations we should have

$$\sin({}_{,}\varphi + {}'\varphi) \,.\, \sin\gamma = 0.$$

So that γ remains undetermined, and ${}_{,}\varphi + {}'\varphi = 180$ or 0, or the observed angles ${}_{,}\varphi$ and ${}'\varphi$, are supplemental to each other, the two equations being only equivalent to one.

If we take the two equations

$$w\,g \cos({}_{,}\theta + \gamma) - l\,m \sin({}_{,}\theta - \delta) = 0,$$
$$w\,g \cos({}'\theta - \gamma) - l\,m \sin({}'\theta - \delta) = 0,$$

and one of the preceding equations,

$$w\,g \cos({}_{,}\varphi + \gamma) + l\,m \sin\delta \cos{}_{,}\varphi = 0,$$

we shall obtain the equations

$$(\cot{}_{,}\theta - \cot{}_{,}\varphi) \,.\, \tan\delta + \cot{}_{,}\varphi \cot\gamma - 1 = 0,$$
$$(\cot{}'\theta + \cot{}_{,}\varphi) \,.\, \tan\delta + \cot{}_{,}\varphi \cot\gamma - 1 = 0:$$

whence

$$\cot{}_{,}\theta - \cot{}'\theta - 2\cot{}_{,}\varphi = 0,$$

giving the relation between the angles ${}_{,}\theta$, ${}'\theta$, and ${}_{,}\varphi$; γ remaining undetermined as before.

all occasions by inverting the poles, the same care and the same precautions must be taken on all occasions, to ensure accuracy, when probably circumstances are most unpropitious.

Although I consider that, in the method I propose, the not changing the magnetism of the needle will be of great advantage in the determination of the dip, a further and still greater advantage which will arise from it will be that we shall, in consequence, obtain a measure of the intensity of terrestrial magnetism. As far as this intensity can be determined by means of an individual needle, the accuracy of the result in all cases depending upon the permanency of the magnetism in the needle, it will be determined from the equations (*c*) and (*d*), by the observations which give the dip, and by this means observations of the most tedious nature will be avoided. In order, however, to attain this object, it is necessary that the centre of gravity of the needle should not be in the centre of figure; and what was considered to constitute perfection in a dipping needle, would render it useless for this purpose; nor must the centre of gravity be in any other point in the magnetic axis of the needle; but the intensity may be determined if it be in any other position, though not equally well from all.

As a dipping needle is an instrument to be employed at places situated on south as well as on the north side of the magnetic equator, it is necessary to have regard to this, in fixing upon a position for the centre of gravity. In order that the position of this point should equally affect the dip on both sides of the equator, it must be in a line at right angles to the axis of the needle and the axis of motion, that is, the angle γ must be 90°; in which case the needle would still be parallel to the horizon at the magnetic equator. The equations (1) and (2) will in this case become

$$w g \sin {}_{,}\theta - l m \sin (\delta - {}_{,}\theta) = 0, \qquad (5)$$

$$w g \sin {}'\theta - l m \sin ({}'\theta - \delta) = 0; \qquad (6)$$

and

$$\tan \delta = \frac{2 \sin {}'\theta \sin {}_{,}\theta}{\sin ({}'\theta + {}_{,}\theta)}, \qquad (g)$$

or

$$\cot \delta = \frac{\cot {}_{,}\theta + \cot {}'\theta}{2}; \qquad (h)$$

also $\dfrac{l m}{w g} = \dfrac{\sin {}_{,}\theta}{\sin (\delta - {}_{,}\theta)}$ . . . (*i*), or $\dfrac{l m}{w g} = \dfrac{\sin {}'\theta}{\sin ({}'\theta - \delta)}$ . . . (*k*)

The principle on which the value of g depends, is not of such ready application as that on which we have determined the value of γ. The magnetic intensity at any place is generally referred to that at the magnetic equator, which is taken as unity. In order, therefore, that this may be the case with any particular needle, we must have

$$l\,m = w\,g, \text{ when } \delta = 0.$$

In this case the equations (5) and (6) become

$$(w\,g + l\,m) \sin {}_{\prime}\theta = 0, \text{ and } (w\,g - l\,m) \,.\, \sin {}^{\prime}\theta = 0.$$

So that ${}_{\prime}\theta = 0$, and ${}^{\prime}\theta$ is indeterminate.

In order, then, that the condition of $\frac{l\,m}{w\,g}$, which is the measure of the intensity, being equal to unity at the magnetic equator, should be fulfilled, it is necessary that the needle, when there, should be horizontal with its centre of gravity downwards, and that it should rest in any position when reversed on its axis of motion. It therefore becomes a question how the position of the centre of gravity of a needle can be adjusted, in a place at a distance from the magnetic equator, so that when the needle is carried there, it shall be horizontal with the centre of gravity downwards, and indifferent to position when reversed on the axis, or so that in this case $\frac{l\,m}{w\,g}$ should be equal to unity. If we knew the law according to which the magnetic intensity varies, this might easily be accomplished; but it is to determine this law that observations are required; and we can only avail ourselves of one which does not very closely accord with observation. This law, deduced from the hypothesis of two magnetic poles near the earth's centre, is that the intensity varies inversely as $\sqrt{(4 - 3\sin^2\delta)}$; from which, if the intensity at the equator is unity, we shall have

$$\frac{l\,m}{w\,g} = \frac{2}{\sqrt{(4 - 3\sin^2\delta)}}$$

and consequently

$$\frac{\sin {}_{\prime}\theta}{\sin(\delta - {}_{\prime}\theta)} = \frac{2}{\sqrt{(4 - 3\sin^2\delta)}}, \text{ and also } \frac{\sin {}^{\prime}\theta}{\sin({}^{\prime}\theta - \delta)} = \frac{2}{\sqrt{(4 - 3\sin^2\delta)}}.$$

From these equations we obtain

$$\cot {}_{\prime}\theta = \cot \delta + \tfrac{1}{2} \sqrt{(4 \cot^2 \delta + 1)},$$
$$\cot {}'\theta = \cot \delta - \tfrac{1}{2} \sqrt{(4 \cot^2 \delta + 1)}.$$

So that knowing the dip at any place, the angles ${}_{\prime}\theta$, ${}'\theta$, which the needle ought to make with the horizon, in order that, according to the assumed law, the condition at the equator should be fulfilled, may be determined.

According to this adjustment of wg to lm, $\cot {}'\theta$ would always be negative, or ${}'\theta$ greater than 90°. It would therefore, in practice, be more convenient to measure the inclination of the needle in this case from the opposite point of the horizon to that from which ${}_{\prime}\theta$ is measured. Taking, then, φ for this inclination, or $\varphi = 180 - {}'\theta$, the equations ($h$) and ($k$) would become

$$\cot \delta = \frac{\cot {}_{\prime}\theta - \cot \varphi}{2}, \qquad \frac{lm}{wg} = \frac{\sin \varphi}{\sin (\delta + \varphi)}.$$

Although the condition of adjusting the centre of gravity in the perpendicular to the two axes is not one of great practical difficulty, yet it becomes difficult when it is also required that, after being magnetized, the needle shall assume certain positions; and it will scarcely admit of accomplishment without some moveable weight being attached to the needle for the purpose of adjustment. To this there is a serious objection, unless the weight can be permanently fixed, after the required adjustment has been effected. But these are practical difficulties which are not insurmountable, and indeed the second condition is not necessary, in order that what is required may be effected with the needle; I shall, however, have occasion to point out a circumstance which will have an influence on that condition, independent of the construction of the needle.

However correct may be the proposed principles of construction, the instrument may fail in its practical object, in consequence of the changes in the angles measured by it being considerably less than corresponding changes in the dip, so that a small error of observation would introduce a considerable error in the result. Although this may, to a certain extent, appear to be the case, yet it is only so when the errors of the corresponding observations tend both the same way; and even then, except in particular situations, the errors are not much increased. Supposing the centre of gravity of the needle to have been adjusted as I have stated, then if the dip is 69° 40′, (nearly that in London at present,) we shall have $_{,}\theta = 45° \ 12'$, $\varphi = 75° \ 52'$, $M = 1{\cdot}7134$; an error $+ 5'$ in $_{,}\theta$, and $- 5'$ in φ, will give the dip 69° 46′, and $M = 1{\cdot}7141$; an error $- 5'$ in $_{,}\theta$, and $+ 5'$ in φ, will give the dip 69° 33′, and $M = 1{\cdot}7128$; an error of $\pm 5'$ in $_{,}\theta$ alone will introduce an error of $\pm 4'$ in the dip; an error of $\pm 5'$ in φ alone will introduce an error of only ∓ 2; and if the errors in $_{,}\theta$ and φ are both plus or both minus, the errors in the dip will be further diminished. An error, then, of 5′ in each of the angles, even when both tend the same way, will only introduce an error of 6′ or 7′ in the dip, and of ·0007 in the intensity. With whatever instruments the dip may be determined, it rarely happens but that the observed angles differ from each other by much more than this; and I doubt whether, taking all circumstances into account, the intensity is ever determined nearer than to the second place of decimals by the method of vibrations.

As the dip decreases, the angle $_{,}\theta$ will decrease, and the angle φ will approximate to 90°; and within two or three degrees of the magnetic equator, φ would be so nearly 90°, that $\cot \delta$ might, without sensible error, be taken equal to $\frac{1}{2} \cot {_{,}\theta}$. If λ is the magnetic latitude, then the formula $\tan \delta = 2 \tan \lambda$ would give $\lambda = {_{,}\theta}$ near to the equator; and in other positions we should have $\cot \lambda = \cot {_{,}\theta} - \cot \varphi$.

I have already mentioned that it had long since occurred to me, to place two needles on the same axis, at a distance from each other, but so as, however, to form one compound needle, and that I had made the calculations from which the dip might be obtained by means of such a needle, without the necessity of inverting the poles; but that I gave up the idea, in consequence of the complicated form of the results, when the positions of the centres of gravity of the needles were undetermined. If, however, the two needles were made of exactly the same weight, and the centre of gravity of each were accurately adjusted in the line perpendicular to the axis of the needle, and so that those centres should be at equal distances from the centre of motion, such a needle would possess many advantages; it would, however, be necessary, for the purposes of adjustment and of more accurate observation, that the instrument should have two graduated circles, one for each needle.

If the needles were placed on the axis, so that their contrary poles should be adjacent, and their axes in the same plane, then, when the centres of gravity of the two needles are on the same side of this plane, the needle should be horizontal; and when on contrary sides, the needle should be indifferent to position. By this means the accuracy of the adjustments might be ascertained.

If the needles were placed on the axis of motion, so that their axes should be at right angles to each other, then they would admit of eight different arrangements with respect to their centres of gravity; four of these having, however, precisely the same reference to one needle which the remaining ones have to the other, or simply arising from reversing the position of the axis of motion. By observing the inclination of the needle in these different arrangements, taking the mean of two corresponding angles as the inclination, we should obtain four equations; and from any two of these, the dip and the ratio of the static momentum of the force of terrestrial magnetism on that of the needle to the static momentum of the needle's weight might be deduced. By taking a mean of the several results, the dip and the measure of the magnetic intensity would be determined, nearly free from errors of adjustment.

In this case, the centre of gravity of each needle will be in the axis of the other. C indicating the centre of the compound needle, M, $M_{,}$, the marked ends of the two needles; G, $G_{,}$, their centres of gravity; let ε be the inclination of C M to the horizon when G is between $M_{,}$ and C, and $G_{,}$ between M and C; θ its inclination when G is between $M_{,}$ and C, and $G_{,}$ is beyond C; φ its inclination when G is beyond the centre, and $G_{,}$ between M and the centre; ψ its inclination when G and $G_{,}$ are both beyond the centre; also $w\,g$ the static momentum of the weight of each needle, and $l\,m$ that of the force of terrestrial magnetism acting on the magnetism of each needle; then the condition of equilibrium will give,

$$w\,g\,(\cos\varepsilon - \sin\varepsilon) - l\,m\,\{\cos(\delta-\varepsilon) - \sin(\delta-\varepsilon)\} = 0 \quad . \quad . \quad . \quad . \quad \text{(I.)}$$
$$w\,g\,(\cos\theta + \sin\theta) + l\,m\,\{\cos(\delta-\theta) - \sin(\delta-\theta)\} = 0 \quad . \quad . \quad . \quad . \quad \text{(II.)}$$
$$w\,g\,(\cos\varphi + \sin\varphi) - l\,m\,\{\cos(\delta-\varphi) - \sin(\delta-\varphi)\} = 0 \quad . \quad . \quad . \quad \text{(III.)}$$
$$w\,g\,(\cos\psi - \sin\psi) + l\,m\,\{\cos(\delta-\psi) - \sin(\delta-\psi)\} = 0 \quad . \quad . \quad . \quad \text{(IV.)}$$

From these the following values of $\cot\delta$ are obtained:

$$\cot\delta = \frac{1-\tan\varepsilon}{1+\tan\theta}; \qquad \cot\delta = \frac{\cot\varepsilon - 1}{\cot\varphi + 1}; \qquad \cot\delta = \frac{(1-\tan\varepsilon).(1-\tan\psi)}{1-\tan\varepsilon.\tan\psi};$$

$$\cot\delta = \frac{1-\tan\psi}{1+\tan\varphi}; \quad \cot\delta = \frac{\cot\psi - 1}{\cot\theta + 1}; \quad \cot\delta = \frac{1-\tan\theta\,.\,\tan\varphi}{(1+\tan\theta)\,.\,(1+\tan\varphi)}.$$

The third and sixth may be put in the forms

$$\cot\delta = \frac{\cos(\varepsilon-\psi)-\sin(\varepsilon+\psi)}{\cos(\varepsilon+\psi)}; \quad \cot\delta = \frac{\cos(\theta+\varphi)}{\cos(\theta-\varphi)+\sin(\theta+\varphi)},$$

somewhat more convenient for computation.

Substituting the mean of the values of δ, thus obtained, each of the equations I., II., III., IV., will give a value of $\frac{l\,m}{\varpi\,g}$; the mean of which values will give the measure of the magnetic intensity.

If the axes of the needles were again brought into the same plane, so that the corresponding poles should be adjacent, then the dip might be obtained either by direct observation, or by computation from the equation (h), according as the centres of gravity of the two needles are on contrary sides, or on the same side of the plane of the axes.

When the centres of gravity are on contrary sides of the plane of the axes, if the adjustment have been correctly made, the centre of gravity of the compound needle will be in the centre of motion, and the observed inclination will be the dip; but if small errors, as must almost necessarily happen, have been made in these adjustments, then, without changing the face of the instrument, four arrangements of the needles upon the axis may be made, and four others by reversing the ends of the axis; and the mean of the inclinations will be the dip, freed from these errors of adjustment. The errors of the instrument itself will, as usual, be counteracted by making corresponding observations with its face reversed.

The axes of the needles being still in the same plane, if the centre of gravity of each be on the same side of that plane, then, by varying the arrangement of the needles and of the axis, four observations may be made with the centre of gravity of the compound needle upwards, and four others with it downwards. From these the dip may be obtained by means of the equation (h), and the intensity by means of the equations (i) and (k), freed from errors of adjustment.

We should thus, with the same needle, obtain the dip by three independent methods, and a measure of the magnetic intensity by two such methods, and their agreement would be a test of the accuracy of the adjustments and of the observations. If a compound needle of such a construction were very accurately made, I consider that it would give accordant and, therefore, very satisfactory results; but, at the same time, I am quite sensible that it would require great care and delicacy in the adjustment of the needle, and in making the observations.

As I am upon the subject of improvements in the dipping needle, it is proper that I should mention one which I had not seen until within a few days, and which has just been completed by Mr. Bate, on a principle suggested, as Captain Beaufort informs me, by Lieutenant Barnett, namely, that the support of the needle should be a knife-edge*. It being necessary that the knife-edge should always have the same position, the needle is moveable on a projecting axis, whose centre is coincident with the edge, and on the other end is fixed a counterpoise to the needle, in the form of a brass needle, which, when horizontal, indicates that the sides of the knife-edge are equally inclined to the agate plane on which it rests, between the magnetized needle and its counterpoise. The knife-edge is to be adjusted so as to pass through the centre of gravity of the whole; and when this is effected, the dip will be obtained without inversion of the poles of the needle. This adjustment must be a matter of great delicacy in the first instance; but if it can be effected, and its permanence ensured, there is every reason to expect that the observations with this needle will not only be more accordant with each other than those obtained with needles supported on cylindrical axes, but that the results will be nearer to the truth.

A considerable defect in this construction, however, is, that the counterpoise acts as dead weight, doubling the inertia of the needle without adding to its power. If a counterpoise simply to the knife-edge were attached to its middle point, the correct position of that edge on the agate planes being determined by a small index above, and the two needles, of the compound needle which I have described, were placed on cylindrical ends, projecting from the knife-edge so that the axes of the cylinders should coincide with it, the needles would act as counterpoises to each other, and with the increase of inertia

* M. Kupffer mentions having made some observations with a needle supported on a knife-edge, but does not describe the instrument. London and Edinburgh Philosophical Magazine and Journal, third series, vol. i. p. 130.

there would be a corresponding increase of power. The adaptation of the knife-edge to the construction which I have described would be a decided improvement; and in this construction the difficulties which must occur in adjusting the knife-edge to the centre of gravity would be avoided. I should therefore propose, if this construction be adopted, that to the knife-edge a counterpoise should be attached, so as to pass freely between the agate planes on which the edge rests. The knife-edge should then be accurately adjusted to the counterpoise, so that the edge should pass through the centre of gravity of the whole. Two needles, of precisely the same form and weight, such as I have before described, are then to be applied to the axes projecting from the knife-edge; and I consider that little practical difficulty would occur in adjusting the centre of gravity of each, very accurately, to the line perpendicular to its axis. The opening in the centre of each needle might be of the form of two truncated cones joined by their smaller ends, which would allow of the axes projecting from the knife-edge being conical, and, at the same time, admit of the needles being reversed on these axes. The two agate planes should be bedded in the same piece of metal, and then worked truly in one plane. The sensibility of such a needle would be much greater than that of any hitherto made, and the utmost delicacy would be required in the adjustments for observation; but if the needle were accurately constructed, and due care were taken in the magnetizing and in making the adjustments and observations, I confidently expect that the dip and intensity would be obtained, by such means, to a degree of certainty hitherto unattained.

The principle, then, on which I propose that a needle should be constructed, whether a single needle, or each needle of the compound needle, is, that it should have an additional weight (a small platinum stud, for instance,) fixed in the line perpendicular both to the axis of the needle and to that of motion, and the whole so adjusted that the centre of gravity be likewise accurately in this line. This additional weight is also to be so adjusted to the magnetism of the needle, that at a place where the needle is horizontal with the weight downwards, (on the magnetic equator,) it shall be nearly indifferent to position when reversed on the axis. Although this condition could only be accurately fulfilled by making these adjustments at the magnetic equator, and even here it would only obtain with an invariable temperature, yet I have shown how such an adjustment may be approximately made at any place where the dip has been determined. Perhaps, for an instrument intended for observation near the magnetic equator, it would be advisable to have two needles, one having the additional weight so adjusted to the needle's magnetism, that at the equator $w\,g$ should be equal to $l\,m$, and the other so that $w\,g$ should be equal to $2\,l\,m$, in order to obviate the degree of indecision that might, in that situation, occur in the direction of the needle; but the necessity or utility of this would be best ascertained by actual observation.

The advantages attending the use of such a needle would be, that the dip would be obtained without reversing the poles of the needle; by which means the result would be not only less liable to error than when that operation is necessary, but the observation would be made in less than half the time usually required: that a measure of the intensity of the terrestrial magnetism would be obtained from the same observations which give the dip: that thus the intensity of the force would not only be obtained by means of the same needle, but also at the same instant as that at which its direction is determined. So that by this means comparative results would be more correctly obtained, and in less time, by one observer, than could be obtained by two, if the intensity were determined by the vibration of a needle. As compared with the single needle, the compound needle which I have described would have the advantage, that with it the adjustments might be verified; that independent results might be obtained; that it would possess greater sensibility, and give more accurate results; and that with it the dip and intensity might be obtained near the magnetic poles, or near the equator, as in other situations. Whether these advantages might not be counterbalanced by the greater nicety required in the adjustments, the longer time for observation, and the greater liability to derangement, can only be determined by actual observation with needles of each construction.

In a paper published in the Philosophical Transactions for 1825, I first investigated the effects which change of temperature produces on the intensity of the magnetism of steel bars, in order to apply the results to the correction of observations which I had made for determining the diurnal changes in the horizontal intensity of terrestrial magnetism. In the same paper I pointed out the necessity of applying such a correction for the difference in the temperatures at which observations of the vibrations of a needle are made, in order that the times of vibration should be reduced to the same temperature*. Although such a correction has since been applied by Professor HANSTEEN, Captain SABINE†, M. KUPFFER‡, M. QUETELET§, and others, yet, as it depends upon the individual needle employed, the application of any formula must be doubtful, unless deduced from experiments with that needle. The method which I have proposed for determining the terrestrial magnetic intensity, would also require that a correction should be applied for the difference in the temperatures at which the observations may be made; and I consider it would be desirable that experiments should be made with the needle, in order to determine the nature of the corrections to be applied, previously to making any observations for the dip or intensity. In the same paper, I have stated, that if a magnet is subjected to a temperature above a certain degree, about 100° FAHR., a portion of the magnetism is destroyed||: I consider that this circumstance might be taken advantage of to adjust the magnetism of the needle to its static momentum, in the ratio required.

It must not be supposed, because I have proposed a very different method, that I am insensible to the advantages to be derived from that proposed by Professor GAUSS, for determining the terrestrial magnetic intensity. I greatly admire the sagacity he has shown in devising means by which an absolute measure of the horizontal force is to be determined; and consider that his methods may, very probably, be advantageously employed in an observatory, where the apparatus remains undisturbed. They may be the means of determining, not only the course of the daily variation of the horizontal force, but the changes which may take place in its intensity in long periods of time. However, although, mathematically considered, the solution of the problem of the determination of the measure of the horizontal force may be complete, yet I foresee many difficulties in the practical application; and without having seen a full account of Professor GAUSS's method, with the experimental details, it is not possible to decide how far these may have been overcome. By the vibrations of a bar A, Professor GAUSS determines the product of the terrestrial

* Philosophical Transactions, 1825, p. 61.

† Philosophical Transactions, 1828, p. 4.

‡ Voyage dans le Caucase, p. 82.

§ Mémoires de l'Académie de Bruxelles, 1830, tom. vi. p. 8.

|| Philosophical Transactions, 1825, p. 63.

intensity by the static momentum of the free magnetism of A. By introducing a second bar B, and by observing at different distances the joint effects of A and of the terrestrial magnetism on B, he proposes to determine the ratio of the terrestrial intensity to the static momentum of the free magnetism of A. Now, if the temperature of A is not the same in the second set of observations as it is in the first, neither will the static momentum of its free magnetism be the same; so that on this account the first observations would give the product of two quantities different from those of which the ratio is obtained from the second; and independently of this source of error, the intensity of terrestrial magnetism is itself changing during the time which must necessarily be employed in making the observations. These considerations make me doubt whether a very accurate measure of the horizontal intensity will be obtained by this method, even in an observatory; and I think it could not, where an apparatus has to be moved from one station to another, be successfully applied, both on this account and on account of the delicacy of the preparatory observations, and of the time requisite for making them, in addition to that required for the observations by which the terrestrial intensity and its variations are to be determined. Some time, I also consider, must elapse before we can expect to find many observers who will be able to obtain, by this method, results from which can be deduced conclusions on whose correctness we can rely. It is likewise to be remembered, that however accurate may be the measure of the terrestrial magnetic force obtained by this method, it is only the resolved part of that force parallel to the horizon which is determined; and that in order to obtain the total force, the object of our research, the dip, determined by some means, must enter as an element. This is also the great objection to the use of Professor Hansteen's apparatus, which is unrivalled for portability and convenience of use: the accomplishment, however, by its means, of the object proposed by Professor Gauss was never contemplated, and is indeed beyond its power, without the application of a second needle.

Royal Military Academy,
April 8th, 1833.